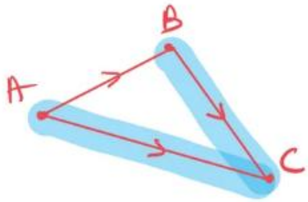
Q1a

a)




$$\overrightarrow{AB} = \overrightarrow{AC} - \overrightarrow{BC}$$

$$= (5\underline{i} - 2\underline{j}) - (-3\underline{i} + k\underline{j})$$

$$= \boxed{8\underline{i} - (2+k)\underline{j}}$$

Q1b

b) $|\overrightarrow{AB}| = \sqrt{8^2 + (2+k)^2} = \sqrt{89}$

$$(2+k)^2 = 89 - 64$$

$$4 + 4k + k^2 = 25$$

$$k^2 + 4k - 21 = 0$$

$$(k+7)(k-3) = 0$$

$$\boxed{k = -7,\ 3}$$

Q2a

a) $\begin{pmatrix} 8 \\ m \end{pmatrix} + \begin{pmatrix} n \\ -2 \end{pmatrix} = \begin{pmatrix} m \\ n \end{pmatrix} - 2\begin{pmatrix} n \\ -2 \end{pmatrix}$

$8 + n = m - 2n$

$m = 3n + 8$ ①

$m - 2 = n + 4$ ②

Sub ① into ②:

$3n + 8 - 2 = n + 4$

$2n = -2$

$n = -1$

Sub $n = -1$ into ①:

$m = 3(-1) + 8 = 5$

$$\boxed{\begin{array}{l} m = 5 \\ n = -1 \end{array}}$$

Q2b

b) $|d| = \sqrt{(2k+1)^2 + (2k-1)^2} = 3k\sqrt{2}$

$(2k+1)^2 + (2k-1)^2 = 18k^2$

$4k^2 + \cancel{4k} + 1 + 4k^2 - \cancel{4k} + 1 = 18k^2$

$8k^2 + 2 = 18k^2$

$2 = 10k^2$

$$\boxed{k = \pm\frac{1}{\sqrt{5}}}$$

Q3a

a) Since $\overrightarrow{OA}$ begins at the origin, A has the coordinates:

$$x = 3k \ , \ y = 5k$$

$$(3k-11)^2 + (5k-7)^2 = 34$$

$$9k^2 - 66k + 121 + 25k^2 - 70k + 49 = 34$$

$$34k^2 - 136k + 136 = 0$$

$$k^2 - 4k + 4 = 0$$

$$(k-2)^2 = 0$$

$$\boxed{k = 2}$$

$$x = 3k = 3(2) = 6 \qquad y = 5k = 5(2) = 10$$

$$\boxed{A\ (6, 10)}$$

Q3b

The point $A$ lies on the circle with equation $(x-11)^2 + (y-7)^2 = 34$. $A$ has position vector $\overrightarrow{OA} = 3k\mathbf{i} + 5k\mathbf{j}$, where $k$ is a constant.

(a) Find the value of $k$, and hence determine the coordinates of $A$. [4]

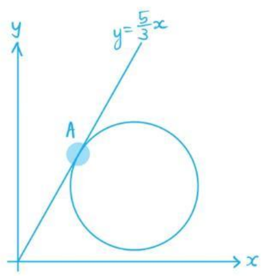
(b) Explain why a line passing through $O$ and $A$ must be a tangent to the circle. [2]

$$m = \frac{\Delta y}{\Delta x} = \frac{5k}{3k} = \frac{5}{3}$$

$$c = 0$$

$$\therefore \ y = \frac{5}{3}x$$

b)




A lies on the line $y = \frac{5}{3}x$. Since there was a solution to the equation in part (a), the line intersects the circle, and because there was only 1 solution, the line must be a tangent.

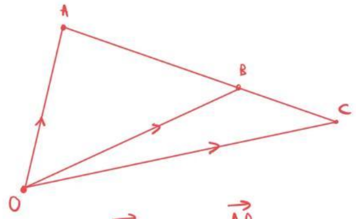
Q4

Points $A$, $B$ and $C$ have position vectors $\overrightarrow{OA} = -6\mathbf{i} - 2\mathbf{j}$, $\overrightarrow{OB} = \mathbf{i} + m\mathbf{j}$ and $\overrightarrow{OC} = 3\mathbf{i} - 8\mathbf{j}$, respectively.

Given that $A$, $B$ and $C$ lie on the same straight line, use a vector method to find the value of $m$.

[5]

$\overrightarrow{AC}$ must be parallel to $\overrightarrow{AB}$. Therefore $\overrightarrow{AC}$ must be a scalar multiple of $\overrightarrow{AB}$. Let's call that scalar multiple $p$.



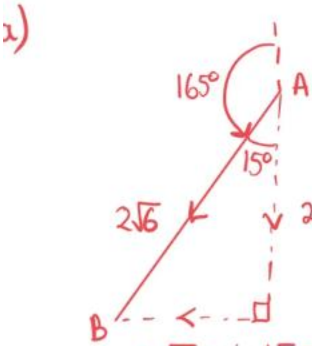

$$\overrightarrow{AC} = p\,\overrightarrow{AB}$$
$$-\overrightarrow{OA} + \overrightarrow{OC} = p\left(-\overrightarrow{OA} + \overrightarrow{OB}\right)$$
$$(-6\underline{i} - 2\underline{j}) + (3\underline{i} - 8\underline{j}) = p\left(-(-6\underline{i} - 2\underline{j}) + (\underline{i} + m\underline{j})\right)$$
$$9\underline{i} - 6\underline{j} = p\left(7\underline{i} + (2+m)\underline{j}\right)$$

$$9 = 7p \qquad p = \frac{9}{7}$$

$$-6 = p(2+m)$$
$$-6 = \frac{9}{7}(2+m)$$
$$-\frac{42}{9} = 2 + m$$
$$\boxed{m = \frac{-20}{3}}$$

Q5A

a)




$$2\sqrt{6}\cos 15 = 3 + \sqrt{3}$$
$$2\sqrt{6}\sin 15 = 3 - \sqrt{3}$$

$$\overrightarrow{AB} = -(3-\sqrt{3})\underline{i} - (3+\sqrt{3})\underline{j}$$
$$= \boxed{(\sqrt{3}-3)\underline{i} - (3+\sqrt{3})\underline{j}}$$

Q5b

b) $|\overrightarrow{AB}| = \sqrt{(\sqrt{3}-3)^2 + (3+\sqrt{3})^2}$

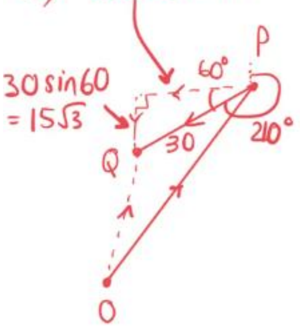
$= 2\sqrt{6}$

unit vector: $\dfrac{\overrightarrow{AB}}{|\overrightarrow{AB}|} = \dfrac{(\sqrt{3}-3)\underline{i} - (3+\sqrt{3})\underline{j}}{2\sqrt{6}}$

$= \boxed{-\dfrac{\sqrt{6}-\sqrt{2}}{4}\underline{i} - \dfrac{\sqrt{6}+\sqrt{2}}{4}\underline{j}}$

Q6a

a) $30\cos 60 = 15$




First leg of journey

$s = 40\text{km/h}$ $\quad t = 1.5\text{h}$

$\overrightarrow{OP} = \underbrace{40(1.5)}_{\text{distance} = s \times t} \boxed{\dfrac{(\underline{i} + 3\underline{j})}{\sqrt{1^2+3^2}}}$

unit vector parallel to direction of travel

$= 6\sqrt{10}\,\underline{i} + 18\sqrt{10}\,\underline{j}$

Second leg of journey

$s = 40\text{km/h}$ $\quad t = 0.75\text{h}$

$|\overrightarrow{PQ}| = 40 \times 0.75 = 30$

Use trig to write $\overrightarrow{PQ}$ in terms of $\underline{i}$ and $\underline{j}$

$\overrightarrow{PQ} = -15\underline{i} - 15\sqrt{3}\,\underline{j}$

$\overrightarrow{OQ} = \overrightarrow{OP} + \overrightarrow{PQ}$

$= 6\sqrt{10}\,\underline{i} + 18\sqrt{10}\,\underline{j} - 15\underline{i} - 15\sqrt{3}\,\underline{j}$

$= \boxed{(6\sqrt{10} - 15)\underline{i} + (18\sqrt{10} - 15\sqrt{3})\underline{j}}$

Q6b

A ship is searching for a radio buoy whose transmitter has ceased functioning. The ship sets out from point $O$ and heads in the approximate direction of the buoy, travelling at a constant speed of 40 km/h in a direction parallel to the vector $\mathbf{i} + 3\mathbf{j}$. After travelling for ninety minutes the ship has reached point $P$. At that time, the ship receives a brief transmission from the buoy indicating that the buoy is at a bearing of 210° from the ship. The ship heads on that bearing at the same constant speed, and reaches the buoy at point $Q$ in another 45 minutes. Given that vector $\overrightarrow{OQ} = x\mathbf{i} + y\mathbf{j}$ km, find the exact values of $x$ and $y$.

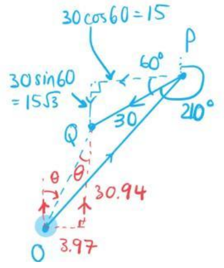
(a) Given that vector $\overrightarrow{OQ} = x\mathbf{i} + y\mathbf{j}$ km, find the exact values of $x$ and $y$.

$(6\sqrt{10} - 15)\underline{i} + (18\sqrt{10} - 15\sqrt{3})\underline{j}$ [7]

(b) How far was the buoy from the ship, and at what bearing, at the time the ship initially left point $O$? Give the distance in kilometers, and give your answers correct to 1 decimal place.

$|\overrightarrow{OQ}|$

[3]




b) $\overrightarrow{OQ} \approx 3.97\underline{i} + 30.94\underline{j}$ (use exact values in calculations!)

$|\overrightarrow{OQ}| = \sqrt{(3.97)^2 + (30.94)^2} = \boxed{31.2\text{ km (1dp)}}$

$\theta = \tan^{-1}\left(\frac{3.97}{30.94}\right) = 7.32°$

Bearing: $\boxed{007.3° \text{ (1dp)}}$

Q7a

a) $|\underline{F}_3| = \sqrt{k^2 + (k\sqrt{3})^2} = 10 = 2k$

$k = 5$

$\underline{F}_3 = 5\underline{i} + 5\sqrt{3}\underline{j}$

$\underline{R} = 7\underline{i} - \underline{j} + x\underline{i} + y\underline{j} + 5\underline{i} + 5\sqrt{3}\underline{j}$

$= 0\underline{i} + 0\underline{j}$

$7 + x + 5 = 0$

$-1 + y + 5\sqrt{3} = 0$

$\boxed{x = -12}$

$\boxed{y = 1 - 5\sqrt{3}}$

Q7b

In an experiment, three forces are acting on a particle. $\mathbf{F}_1 = 7\mathbf{i} - \mathbf{j}$ newtons and $\mathbf{F}_2 = x\mathbf{i} + y\mathbf{j}$ newtons are both constant forces, although the values of $x$ and $y$ are initially unknown. The third force is $\mathbf{F}_3 = k\mathbf{i} + k\sqrt{3}\mathbf{j}$ newtons, where $k \geq 0$ is a parameter that can be varied by the experimenters. The resultant force $\mathbf{R}$ acting on the particle is given by $\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$.

(a) Given that $\mathbf{R} = \mathbf{0}$ when the magnitude of $\mathbf{F}_3$ is 10 newtons, find the exact values of $x$ and $y$.

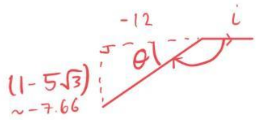
$x = -12 \qquad y = (1 - 5\sqrt{3})$ $-7.66\ldots$

[4]

(b) Find the magnitude of $\mathbf{F}_2$ and the angle it makes with the vector $\mathbf{i}$. Give your answers correct to 1 decimal place.

b) $|\underline{F_2}| = \sqrt{x^2+y^2} = \sqrt{(-12)^2 + (1-5\sqrt{3})^2}$

$= \boxed{14.2 \text{ Newtons (1dp)}}$



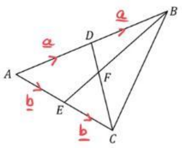

$\alpha = 180° - \theta$

$= 180 - \tan^{-1}\left(\frac{7.66\ldots}{12}\right) = \boxed{147.4° \text{ (1dp) clockwise from } \underline{i}}$

Q8A

## Question 8a

In triangle $ABC$, $D$ is the midpoint of $AB$ and $E$ is the midpoint of $AC$. $BE$ and $CD$ intersect at point $F$.




(a) Given that $\overrightarrow{AB} = 2\mathbf{a}$ and $\overrightarrow{AC} = 2\mathbf{b}$, write the vectors $\overrightarrow{BC}$, $\overrightarrow{BE}$ and $\overrightarrow{CD}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

[3

a) $\overrightarrow{BC} = -\overrightarrow{AB} + \overrightarrow{AC} = \boxed{-2\underline{a} + 2\underline{b}}$

$\overrightarrow{BE} = -\overrightarrow{AB} + \overrightarrow{AE} = \boxed{-2\underline{a} + \underline{b}}$

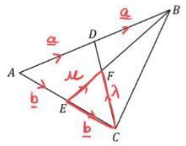
$\overrightarrow{CD} = -\overrightarrow{AC} + \overrightarrow{AD} = \boxed{-2\underline{b} + \underline{a}}$

(b) By setting up and solving a suitable vector equations, prove that each of $BE$ and $CD$ divides the other in the ratio 1: 2

[6

Q8b

In triangle $ABC$, $D$ is the midpoint of $AB$ and $E$ is the midpoint of $AC$. $BE$ and $CD$ intersect at point $F$.




(a) Given that $\overrightarrow{AB} = 2\mathbf{a}$ and $\overrightarrow{AC} = 2\mathbf{b}$, write the vectors $\overrightarrow{BC}$, $\overrightarrow{BE}$ and $\overrightarrow{CD}$ in terms of **a** and **b**.

$\overrightarrow{BC} = -2\underline{a} + 2\underline{b}$, $\overrightarrow{BE} = -2\underline{a} + \underline{b}$, $\overrightarrow{CD} = -2\underline{b} + \underline{a}$ [3]

(b) By setting up and solving suitable vector equations, prove that each of $BE$ and $CD$ divides the other in the ratio 1: 2.

[6]

Let's write the vector $\overrightarrow{CF}$ in two different ways, then equate the coefficients and solve the simultaneous equations that result.

b) $\overrightarrow{EF} = \mu \overrightarrow{EB}$ $\qquad$ $\overrightarrow{CF} = \lambda \overrightarrow{CD}$

$= \mu(2\underline{a} - \underline{b})$ $\qquad$ $= \lambda(-2\underline{b} + \underline{a})$

$\overrightarrow{CF} = \overrightarrow{CE} + \overrightarrow{EF}$

$\lambda(-2\underline{b} + \underline{a}) = -\underline{b} + \mu(2\underline{a} - \underline{b})$

$-2\lambda\underline{b} + \lambda\underline{a} = (-1-\mu)\underline{b} + 2\mu\underline{a}$

$-2\lambda = -1 - \mu$ $\qquad$ $\lambda = 2\mu$

$-2(2\mu) = -1 - \mu$

$3\mu = 1$

$\mu = \frac{1}{3}$

$\lambda = 2\left(\frac{1}{3}\right) = \frac{2}{3}$

So each of BE and CD divides the other in the ratio 1:2.